GRAPH THEORY

Tutorial – 7

1. Suppose that κ(G) = k and diam G = d. Prove that n(G) ≥ k(d-1)+2 and $\alpha(G) \geq \lceil (1+d)/2 \rceil$.

2. Let G be a k-connected graph, and let S,T be disjoint subsets of V(G) with size at least k. Prove that G has k pairwise disjoint S,T-paths.

3. Prove that if G is 2-connected, then G–xy is 2-connected if and only if x and y lie on a cycle in G–xy.

4. Let X and Y be disjoint sets of vertices in a k-connected graph G. Let $u(x)$ for $x \in X$ and $w(y)$ for $y \in Y$ be nonnegative integers such that $\sum_{x \in X} u(x) = \sum_{y \in Y} w(y) = k$. Prove that G has k pairwise internally disjoint X,Y-paths so that $u(x)$ of them start at x and $w(y)$ of them end at y, for $x \in X$ and $y \in Y$.

5. For $S \subseteq V(G)$, let $d(S) = |[S,\bar{S}]|$. Let X and Y be nonempty proper vertex subsets of G. Prove that $d(X \cap Y) + d(X \cup Y) \le d(X) + d(Y)$.